CORRIGENDUM: BRIANÇON-SPEDER EXAMPLES AND THE FAILURE OF WEAK WHITNEY REGULARITY

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ABSTRACT. We correct statements in our paper about the weak Whitney regularity of certain examples of Briançon and Speder. As a consequence, it follows from the calculations in our paper that weak Whitney regularity is not equivalent to Whitney regularity for families of complex hypersurfaces with isolated singularities.

Terry Gaffney pointed out to us a mistake in our paper [1] which affects the conclusion. Instead of showing that the famous Briançon-Speder examples are not weakly Whitney regular, it turns out that our calculations show that all of these examples are indeed weakly Whitney regular, although not Whitney regular, thus answering negatively the question as to whether Whitney regularity and weak Whitney regularity are equivalent for complex analytic varieties.

The mistake first occurs at the foot of page 94 and on page 95 of [1] where we take the gradient of the function $F : (\mathbb{C}^3 \times \mathbb{C}, 0^3 \times \mathbb{C}) \to (\mathbb{C}, 0)$ at a point $(x, y, z, t) \in \mathbb{C}^3 \times \mathbb{C}$, with $F(x, y, z, t) = x^5 + txy^6 + y^7z + z^{15}$. Instead of taking $\operatorname{grad}_x F$ (using the notation of our paper) to define the orthogonal vector to the tangent space of the hypersurface, we must take $\operatorname{grad}_x F$ (the complex conjugate). It then follows that the condition on the curves

$$\begin{array}{rcl} x(s) & = & s^8, \\ y(s) & = & as^5, \\ z(s) & = & \frac{4}{a^7}\lambda s^5, \\ t(s) & = & -\frac{5}{a^6}s^2, \end{array}$$

with $a \neq 0$, for weak Whitney regularity to fail, is that the complex constant a satisfies

$$a^8 \overline{a}^8 = -8$$
, i.e., $|a|^{16} = -8$.

As the lefthand side is a positive real number, there is no solution to this equation and it follows from the calculations in our paper that weak Whitney regularity holds on all curves at the origin, giving the first example known distinguishing between weak Whitney and Whitney regularity for complex analytic stratifications.

A similar correction applies to the argument on page 100 and the argument in Section 9 on page 106 concerning the other Briançon-Speder examples. Thus for these examples also, weak Whitney regularity holds, while Briançon and Speder showed that Whitney regularity fails [2].

We thank Terry Gaffney for showing us the mistake in [1].

WEAK WHITNEY REGULARITY

References

- Bekka, K. and Trotman, D., Briançon-Speder examples and the failure of weak Whitney regularity, Journal of Singularities 7 (2013), 88-107.
- [2] Briançon, J. and Speder, J.-P., La trivialité topologique n'implique pas les conditions de Whitney, C. R. Acad. Sci. Paris Sér. A-B 280 (1975), 365-367.

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