Errata of "The fundamental group of the complement of the singular locus of Lauricella's F_C"

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1. Lemma 3.4 and the proof of (3.2)

Lemma 3.4 should be the following.

Lemma 3.4. *The loop $\ell_{1\cdots10\cdots00\cdots0}$ in $\pi_1(P(0),\varepsilon')$ changes into*

$$\left(\prod^{\prec}_{(a_1,\dots,a_{p+q-1})\in I_{p,q}} \ell_{a_1\cdots a_{p+q-1}0\cdots0}\right)\ell_{1\cdots10\cdots01\cdots0}\left(\prod^{\prec}_{(a_1,\dots,a_{p+q-1})\in I_{p,q}} \ell_{a_1\cdots a_{p+q-1}0\cdots0}\right)^{-1}$$

in $\pi_1(P(1),\varepsilon')$, where we set

$$I_{p,q}=\{(a_1,\dots,a_{p+q-1})\mid (1\dots,\overset{p-1}{1},0,\dots,0)\preceq(a_1,\dots,a_{p+q-1})\preceq(1,\dots,1)\},$$

and the notation $\prod^{\prec}$ means the product multiplying in ascending order of indices with respect to $\prec$.

The proof of (3.2) in pp.301–302 is also changed as follows. (We do not write claims which are not changed.)

Fact 3.1 and Lemma 3.4 imply

$$\begin{aligned}
&(\gamma_1\cdots\gamma_{p-1})\gamma_0(\gamma_1\cdots\gamma_{p-1})^{-1}\\
&=\gamma_{p+q}\left(\prod^{\prec}_{(a_1,\dots,a_{p+q-1})\in I_{p,q}}(\gamma_1^{a_1}\cdots\gamma_{p+q-1}^{a_{p+q-1}})\gamma_0(\gamma_1^{a_1}\cdots\gamma_{p+q-1}^{a_{p+q-1}})^{-1}\right)\\
&\quad\cdot(\gamma_1\cdots\gamma_{p-1}\gamma_{p+q})\gamma_0(\gamma_1\cdots\gamma_{p-1}\gamma_{p+q})^{-1}\\
&\quad\cdot\left(\prod^{\prec}_{(a_1,\dots,a_{p+q-1})\in I_{p,q}}(\gamma_1^{a_1}\cdots\gamma_{p+q-1}^{a_{p+q-1}})\gamma_0(\gamma_1^{a_1}\cdots\gamma_{p+q-1}^{a_{p+q-1}})^{-1}\right)^{-1}\gamma_{p+q}^{-1}.
\end{aligned}$$

Note that the first factor of $\prod^{\prec}$ is $(\gamma_1\cdots\gamma_{p-1})\gamma_0(\gamma_1\cdots\gamma_{p-1})^{-1}$. Multiplying $(\gamma_1\cdots\gamma_{p-1})\gamma_0^{-1}(\gamma_1\cdots\gamma_{p-1})^{-1}\gamma_{p+q}^{-1}$ by left and $\gamma_{p+q}(\cdots)$ by right, we obtain

$$\begin{aligned}
(3.3)\quad &(\gamma_1\cdots\gamma_{p-1})\gamma_0^{-1}(\gamma_1\cdots\gamma_{p-1})^{-1}\gamma_{p+q}^{-1}(\gamma_1\cdots\gamma_{p-1})\gamma_0(\gamma_1\cdots\gamma_{p-1})^{-1}\cdot\gamma_{p+q}\\
&\cdot(\gamma_1\cdots\gamma_{p-1})\gamma_0(\gamma_1\cdots\gamma_{p-1})^{-1}\\
&\cdot\left(\prod^{\prec}_{(a_1,\dots,a_{p+q-1})\in I'_{p,q}}(\gamma_1^{a_1}\cdots\gamma_{p+q-1}^{a_{p+q-1}})\gamma_0(\gamma_1^{a_1}\cdots\gamma_{p+q-1}^{a_{p+q-1}})^{-1}\right)\\
&=\left(\prod^{\prec}_{(a_1,\dots,a_{p+q-1})\in I'_{p,q}}(\gamma_1^{a_1}\cdots\gamma_{p+q-1}^{a_{p+q-1}})\gamma_0(\gamma_1^{a_1}\cdots\gamma_{p+q-1}^{a_{p+q-1}})^{-1}\right)\\
&\quad\cdot(\gamma_1\cdots\gamma_{p-1}\gamma_{p+q})\gamma_0(\gamma_1\cdots\gamma_{p-1}\gamma_{p+q})^{-1},
\end{aligned}$$

where we set $I'_{p,q}=I_{p,q}-\{(1\dots,\overset{p-1}{1},0,\dots,0)\}$. We prove Theorem 1.2 by using this equality.

Proof of Theorem 1.2. We show the theorem by induction on $p+q \geq 2$. As mentioned in Remark 3.3, it is sufficient to show (3.2) for each p, q.

First, we show the case $p+q=2$. In this case, we have only to show (3.5). The equality (3.3) for $p=q=1$ is

$$1 \cdot \gamma_0^{-1} \cdot 1 \cdot \gamma_2^{-1} \cdot 1 \cdot \gamma_0 \cdot 1^{-1} \cdot \gamma_2 \cdot 1 \cdot \gamma_0^{-1} \cdot 1 \cdot (\gamma_1 \gamma_0 \gamma_1^{-1}) = \gamma_1 \gamma_0 \gamma_1^{-1} \cdot \gamma_2 \gamma_0 \gamma_2^{-1}.$$

By (3.4), the left-hand side equals to

$$\gamma_0^{-1} \gamma_2^{-1} \gamma_0 \gamma_2 \gamma_0 \gamma_1 \gamma_0 \gamma_1^{-1} = \gamma_0^{-1} \gamma_0 \gamma_2 \gamma_0 \gamma_2^{-1} \gamma_1 \gamma_0 \gamma_1^{-1} = \gamma_2 \gamma_0 \gamma_2^{-1} \cdot \gamma_1 \gamma_0 \gamma_1^{-1}.$$

Thus (3.5) is proved.

Next, we assume that we have proved (3.2) for any p, q with $p+q \leq k-1$ (recall Lemma 3.5), and prove (3.2) in the case $p+q=k$.

By applying Claim 3.6 to the right-hand side of (3.3), we have

(3.6)

$$\begin{aligned}
&(\gamma_1 \cdots \gamma_{p-1})\gamma_0^{-1}(\gamma_1 \cdots \gamma_{p-1})^{-1}\gamma_{p+q}^{-1}(\gamma_1 \cdots \gamma_{p-1})\gamma_0(\gamma_1 \cdots \gamma_{p-1})^{-1} \cdot \gamma_{p+q} \cdot (\gamma_1 \cdots \gamma_{p-1})\gamma_0(\gamma_1 \cdots \gamma_{p-1})^{-1} \\
&\cdot \left(\prod^{\prec}_{(1\ldots,1,0,\ldots,0) \prec (a_1,\ldots,a_{p+q-1}) \prec (0,\ldots,0,1,\ldots,1)} (\gamma_1^{a_1} \cdots \gamma_{p+q-1}^{a_{p+q-1}})\gamma_0(\gamma_1^{a_1} \cdots \gamma_{p+q-1}^{a_{p+q-1}})^{-1} \right) \\
&\cdot (\gamma_p \cdots \gamma_{p+q-1})\gamma_0(\gamma_p \cdots \gamma_{p+q-1})^{-1} \\
&= \left(\prod^{\prec}_{(1\ldots,1,0,\ldots,0) \prec (a_1,\ldots,a_{p+q-1}) \prec (0,\ldots,0,1,\ldots,1)} (\gamma_1^{a_1} \cdots \gamma_{p+q-1}^{a_{p+q-1}})\gamma_0(\gamma_1^{a_1} \cdots \gamma_{p+q-1}^{a_{p+q-1}})^{-1} \right) \\
&\quad \cdot (\gamma_p \cdots \gamma_{p+q-1})\gamma_0(\gamma_p \cdots \gamma_{p+q-1})^{-1} \cdot (\gamma_1 \cdots \gamma_{p-1}\gamma_{p+q})\gamma_0(\gamma_1 \cdots \gamma_{p-1}\gamma_{p+q})^{-1}.
\end{aligned}$$

We rewrite the first line:

(3.7)

$$\begin{aligned}
&(\gamma_1 \cdots \gamma_{p-1})\gamma_0^{-1}(\gamma_1 \cdots \gamma_{p-1})^{-1}\gamma_{p+q}^{-1}(\gamma_1 \cdots \gamma_{p-1})\gamma_0(\gamma_1 \cdots \gamma_{p-1})^{-1} \cdot \gamma_{p+q} \cdot (\gamma_1 \cdots \gamma_{p-1})\gamma_0(\gamma_1 \cdots \gamma_{p-1})^{-1} \\
&= (\gamma_1 \cdots \gamma_{p-1})\gamma_0^{-1}\gamma_{p+q}^{-1}\gamma_0\gamma_{p+q}\gamma_0(\gamma_1 \cdots \gamma_{p-1})^{-1} \\
&= (\gamma_1 \cdots \gamma_{p-1}) \cdot \gamma_{p+q}\gamma_0\gamma_{p+q}^{-1} \cdot (\gamma_1 \cdots \gamma_{p-1})^{-1}.
\end{aligned}$$

The left-hand side of (3.6) is

(3.8)

$$\begin{aligned}
&(\gamma_1 \cdots \gamma_{p-1}\gamma_{p+q})\gamma_0(\gamma_1 \cdots \gamma_{p-1}\gamma_{p+q})^{-1} \\
&\cdot \left(\prod^{\prec}_{(1\ldots,1,0,\ldots,0) \prec (a_1,\ldots,a_{p+q-1}) \prec (0,\ldots,0,1,\ldots,1)} (\gamma_1^{a_1} \cdots \gamma_{p+q-1}^{a_{p+q-1}})\gamma_0(\gamma_1^{a_1} \cdots \gamma_{p+q-1}^{a_{p+q-1}})^{-1} \right) \\
&\cdot (\gamma_p \cdots \gamma_{p+q-1})\gamma_0(\gamma_p \cdots \gamma_{p+q-1})^{-1}.
\end{aligned}$$

By Claim 3.6, $(\gamma_1 \cdots \gamma_{p-1}\gamma_{p+q})\gamma_0(\gamma_1 \cdots \gamma_{p-1}\gamma_{p+q})^{-1}$ commutes with the second line. Therefore, (3.6) implies the commutativity (3.2). □

Note that we do not use Claim 3.7.

2. Typos

- p.297, Remark 2.1:
  The loop γ_k $(1 \leq k \leq n)$ turns the hyperplane $(x_k = 0)$, and γ_0 turns the hypersurface $S^{(n)}$ around the point $\frac{1}{n^2} \cdot \mathbf{1}$, positively. Note that $\frac{1}{n^2} \cdot \mathbf{1}$ is the nearest to the origin in $S^{(n)} \cap (x_1 = x_2 = \cdots = x_n) = \left\{ \frac{1}{n^2} \cdot \mathbf{1}, \frac{1}{(n-2)^2} \cdot \mathbf{1}, \dots \right\}$.
- p.299, above Figure 3:
  (a) $t^{(1)}_{a_1 \cdots a_{k-1} 0 a_{k+1} \cdots a_{n-1}} = t^{(0)}_{a_1 \cdots a_{k-1} 1 a_{k+1} \cdots a_{n-1}}, t^{(1)}_{a_1 \cdots a_{k-1} 0 a_{k+1} \cdots a_{n-1}} = t^{(0)}_{a_1 \cdots a_{k-1} 1 a_{k+1} \cdots a_{n-1}}$,
  (b) $t^{(\theta)}_{a_1 \cdots a_{k-1} 1 a_{k+1} \cdots a_{n-1}}$ moves in the upper half-plane,
  (c) $t^{(\theta)}_{a_1 \cdots a_{k-1} 0 a_{k+1} \cdots a_{n-1}}$ moves in the lower half-plane.
- p.299, line -2:
  $(r_1, \dots, e^{2\pi\sqrt{-1}\theta} r_k, \dots, r_{n-1}, \varepsilon') \in L(\theta)$.
- p.300, line 1:
  $\ell_{a_1 \cdots a_{k-1} 1 a_{k+1} \cdots a_{n-1}}$
- p.300, Fact 3.1:
  $\eta(\ell_{a_1 \cdots a_{k-1} 1 a_{k+1} \cdots a_{n-1}}) = \gamma_k \cdot \eta(\ell_{a_1 \cdots a_{k-1} 0 a_{k+1} \cdots a_{n-1}}) \cdot \gamma_k^{-1}$

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