Proceedings of the Workshop

Singularities in Aarhus

in honor of

Andrew du Plessis

on the occasion
of his sixtieth birthday

17-21 August 2009

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Contents

This volume contains proceedings of the international workshop *Singularities in Aarhus* held in honor of Andrew du Plessis to celebrate his sixtieth birthday. The workshop took place at the Department of Mathematical Sciences of Aarhus University, Denmark in the week of August 17-21, 2009. Its main theme was singularity theory, both of varieties and mappings. The meeting was attended by about sixty participants from all over the world.

The papers in this volume cover a variety of subjects discussed at the workshop. All the manuscripts have been carefully peer-reviewed. We would like to express our gratitude to the authors for their contributions as well as to the referees for the high quality job.

We also thank all the participants – especially the speakers – who made the meeting successful and fruitful. Last but not least, we are very grateful for the financial support received from the Department of Mathematical Sciences of Aarhus University, from the grant "Symmetry and Moduli Problems in Topology" allocated by the Danish Agency for Science, Technology and Innovation, and from the Center for Topology and Quantization of Moduli spaces (CTQM). The CTQM funding was allocated from the Niels Bohr Visiting Professorship Grant provided by the Danish National Research Foundation.

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The editors

Andrew du Plessis: the story so far

It is a great pleasure to celebrate the 60th birthday of Andrew, my long time friend and collaborator. My personal association with Andrew goes back to 1970, when he arrived in Liverpool as my research student, having just completed a first degree at Cambridge. Andrew's father was also a mathematician, an analyst, then at the University of Newcastle.

Andrew's arrival coincided with the end of our year long Liverpool Singularities Symposium. Among the striking new developments reported that year (by Haefliger) was a technique due to Gromov, dubbed 'homotopy integration', for constructing examples of geometric structures.

Andrew set to work to apply this new idea to problems in singularity theory, and in due course wrote an excellent thesis doing this, which led to his first 3 publications $[1, 2, 4]^{1}$. During this period we had close contact, and I came to regard Andrew as friend and collaborator more than just as student, with several common interests.

When his SERC grant ran out, Andrew obtained a research assistantship at Bangor. This conveniently allowed him time to complete writing up his work, to explore the mountains of Snowdonia, and also to visit Liverpool every couple of weeks to participate in our Singularities Seminar. It was a particularly noteworthy seminar that year, working through a proof of Mather's topological stability theorem, and led by Eduard Looijenga: and a year in which we all learned a lot. The final notes [3] of the seminar remain a key reference in this whole area.

From Bangor, Andrew moved (in 1977) to Aarhus. I was told later that within his first 6 months he had explored the life of the city and had learned to speak, and to lecture in, fluent Danish; and it was fairly soon that he and Annie got together. Perhaps understandably, there is a slight gap in his publications at this point.

But then he began a wonderfully productive period, with a series of great ideas. His next paper [5] obtained the first effective estimates of orders of determinacy of map-germs for right-left equivalence. The techniques were developed and extended in later papers of Andrew and collaborators [7, 12, 18], and led to effective classifications of germs of low codimensions, several of which were published. Unfortunately, the lists available now seem shorter than those that existed 25 years ago: some may still be buried in piles of paper in Andrew's office.

In his paper [6], Andrew made ingenious use of known methods to develop a new technique to study the family of maps with a fixed $k-$jet: here he proved that all germs except for those in a subset of infinite codimension are topologically finitely determined.

In [8] he found the conditions ('semi-nice dimensions') necessary and sufficient for map-germs to be finitely $C^{\infty}-$determined (for right-left equivalence) in general, and extended this in [10] to a global result. Outside these dimensions, he gives a map not homotopic to a $C^{\infty}-$stable map, and even one not homotopic to a map with all germs finitely determined. He also combined this with his own early work to find in favourable cases sufficient conditions. In [13] these ideas are extended to give general results for $C^{1}-$stability (nice dimensions) and for finite and even for $\infty- C^{1}-$determinacy (semi-nice dimensions).

Next he began a collaboration with Leslie Wilson and others producing a series of beautiful papers on right equivalence [11, 14, 16, 19], showing (under mild conditions on f) that:

f is $J_f^2 - \mathcal{R}-$determined,

f is determined up to $\mathcal{R}-$equivalence by Σ_f and $f|\Sigma_f$,

the group of $\mathcal{R}-$symmetries of f is homotopically trivial,

$f|\Sigma_f$ is a normalisation of Δ_f, and hence:

f is determined up to right equivalence by Δ_f.

This suggests a big challenge of finding reasonable conditions under which the homeomorphism

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[1]The references are to the list of Andrew's publications which folllows.

type of Δ_f determines f up to topological right equivalence.

Andrew continued thinking about topological stability, and in the mid 1980's came up with a brilliant idea (disruptive germ classes) for obtaining necessary conditions for stability. This began our period of close collaboration, which lasted about a dozen years and led to our book [17] on stability. It was a most enjoyable period, exciting mathematically, with congenial companionship, (ir)regular meetings at exotic locations, and of course numerous visits to each other at Aarhus and Liverpool.

The usual pattern was that Andrew and I would talk together, often seeking a way round a problem, then separate and each try to write something, then discuss what we had written. When we were not together Andrew would rarely answer letters promptly, but would then send a huge package of handwritten manuscript which I would write (or later type) out, editing and modifying it as I went.

At first we had planned a series of related papers: on the whole, I was doing classifications, Andrew was producing geometrical ideas, and I was typing them up. But once Andrew had built on Jim Damon's ideas to obtain a more general argument for sufficiency, it was clear we should put the work together as a book. The process had its frustrations: every time I thought we had finished and could send the manuscript off for publication, Andrew came up with another brilliant idea, which took one or two years to write up, and added a hundred pages to the length of the manuscript. The book took nearly all our research output for 10 years. Filling in extra points, and finding a number of applications of the book's results or ideas, led to several more years' work and numerous papers: [15] was an advance summary, papers [20, 24, 25, 28, 32] all arise directly from topics in the book; [30] is an application of the main result, and another idea of Andrew's led to the sequence [21-23, 26, 27, 29, 35-38].

I must mention also Andrew's more recent collaboration [31, 33] with David Trotman, with work on stratified transversality, and on a tantalising conjecture that would resolve a number of problems and strengthen the main results in the book; and there are other significant projects at various stages of completion.

I conclude with my very best wishes to Andrew for the future.

Terry Wall

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