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## A note on Goo Ishikawa

Goo (Go-o) Ishikawa is a well known Japanese member of the international community of Singularity Theory. Over three decades, he has been running on the top front in his research fields, especially with focusing on geometry of singular mappings equipped with certain differential systems. It creates a new bridge between Singularity Theory and Differential Geometry (and its various applications).

He was born on November 2, 1957 in Fukushima, Japan, and grew up there. Afterwards, he entered Kyoto University and there he was fascinated by the beauty of mathematics. Around 1980, he began to study Singularity Theory of Mappings, which was one of the most hottest topics in that time – first he met J. Mather's fundamental papers and V. I. Arnol'd's attractive works related to symplectic/contact geometry, and perhaps those must have been merged into a 'kernel' of Goo's mathematics later. His advisor was Masahisa Adachi, who regularly organized "Differential Topology Seminar" at Kyoto University, and Goo was a main contributor. Many people gathered for this seminar, e.g., Shyuichi Izumiya, Masahiro Shiota, Shuzo Izumi, Satoshi Koike and Isao Nakai. In 1985, he got PhD at Kyoto University and began his first career at Nara Women's University. Three years after, he moved to Hokkaido University. Since then, he has been working surrounded by the beautiful nature of the northern earth.

When he was a PhD student, his handwriting seminar note on Hilbert's 16<sup>th</sup> problem was widely circulated in topology community in Japan, and actually this became the theme of his PhD thesis. "The number of singular points in a pencil of real plane algebraic curves" (1985). On the other hand, he also worked on sheaves of  $C^{\infty}$ -rings, influenced by works of Malgrange, Tougeron and others – his first original paper, Families of functions dominated by distributions of C-classes of mappings, has been published in Ann. Inst. Fourier (1983), in which he introduced the notion of ramification modules. This notion took an important role at Goo's long-term project. He then started to explore singularities of tangent developables of curves in  $\mathbb{R}^n$  in relation with the theory of singular Lagrange and Legendre singularities; here a typical singularity is of type open swallowtail. Also he studied, with his own techniques, singular Lagrange immersions having typical singularities named open Whitney umbrellas. The theory of opening of map-germs, introduced later by Goo himself, provides a unified method for characterizing those new important classes of singularities arising in various geometric applications. Indeed, Goo's attempt was to establish a Mather-type framework for a new classification theory of map-germs having integrability on certain differential systems. That is truly his own original theory and it has been quite successful – for example, its application has matured into the theory of *frontals* and *tangential mappings*. As for such kinds of classification problems, Goo produced several joint works especially with S. Janeczko, and also with I. Bogaevsky, A. Davydov, L. Wilson, H. Brodersen, etc. and with Japanese co-workers. For instance, Goo and Janeczko established a symplectic classification of plane curves, and Goo together with Y. Machida and M. Takahashi studied tangent surfaces in detail from the viewpoint of special geometry, e.g.,  $D_4$ -geometry, and so on. Besides, in an earlier period (1987), Goo and Takuo Fukuda published a joint paper which provides a new algebraic formula for counting the number of cusps appearing in a generic perturbation of a given finite real and complex plane-to-plane map-germ. That was influential in two-folded ways; their formula in complex case was soon generalized by several authors into the case of higher dimension for Thom-Boardman singularities, and real enumerations using the Eeisenbud-Levine theorem attracted several younger people to find a new research direction.



As known, Goo and his elder colleague and old friend, Shyuichi Izumiya, created "Sapporo School" in Singularity Theory – they have organized many conferences, raised many students, and especially, in 1998, they published a graduate course textbook entitled with Applied Singularity Theory (Ohyo-Tokuiten-ron), which was the first comprehensive book written in Japanese on Lagrange and Legendre singularity theory and applications. In 1994-1995, Goo visited the University of Liverpool as his sabbatical hosted by C.T.C. Wall. This experience has led to a deep and widespread development of his own research, resulting in many international collaborative researches and warmest friendships with foreign researchers. Since then, he has organized several international symposiums together with Shvuichi, including the 12<sup>th</sup> International Research Institute of the Mathematical Society of Japan "Singularity Theory and Its Application" at Sapporo (2003), "Japanese-Polish working days" with S. Janeczko, Japanese-Russia bilateral project with A. Davidov and I. Bogaevsky, and so on. Also he has frequently been invited to Scientific Committees and to give keynote/plenary talks at many international conferences around the world.

On a broad range of topics, Goo Ishikawa has supervised more than five PhD students, e.g., T. Yamamoto, T. Fukunaga, W. Yukuno, A. Tsuchida, T. Yamashita, and has had 73 publications together with 26 co-authors (according to MathSciNet). He has written totally 13 books so far – there is one lecture note in English, *Singularities of Curves and Surfaces in Various Geometric Problems*, CAS Lecture Notes 10, Exact Sciences (2015), and three advanced textbooks were written in Japanese with several co-authors, e.g., *Applied Singularity Theory* mentioned above. There are five textbooks for undergraduate courses on linear algebra, calculus, sets and logic, topology, and four enlightenment booklets for general public readers, one of which is a lovely collection of his witty answers to students' funny questions on mathematics and life (this booklet has received positive ratings in reviews on amazon !).

Goo is still quite active on researches in mathematics. We wish you a happy birthday Goo, sincerely from all your friends and colleagues, and look forward to working with you for many years to come !